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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2019
SECOND YEAR (BATCH 2017-20)
MATH FOR INDUSTRIAL CHEMISTRY (General)
Paper : IV

Date : 28/05/2019
Time : 11.00 am – 2.00 pm
Full Marks : 75

[Use a separate Answer Book for each group]

Group-A

***Answer any four questions* from questions nos. 1 to 6:** [4×5]

1. a) Test for convergence : $\int_1^\infty \frac{x\,dx}{(1+x)^3}$. 3

   b) Evaluate : $\int_{-\infty}^\infty \frac{dx}{1+x^2}$. 2

2. a) Test the convergence : $\int_1^\infty \frac{\sin^2 x}{x^2}dx$. 3

   b) Evaluate : $\int_0^{\frac{\pi}{2}} \cos^4 x\,dx$. 2

3. Show that the area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$

4. Find $\iint_R (x^2 + 2xy)\,dx\,dy$ where *R* is region bounded by the curves $y = x$ and $y = x^2$.

5. Find the area of the surface of revolution generated by revolving the arc of the parabola $x^2 = 4ay$ bounded by the latus rectum about the y-axis .

6. Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the y-axis .

***Answer any two questions* from questions nos. 7 to 10** [2×5]

7. a) Show that $\vec{\nabla}\cdot\vec{\nabla}f = 40$ at $(1,1,1)$, where $f = 2x^3y^2z^4$. 2

   b) If $\vec{a}$ be a constant vector , prove that $\text{curl}(\vec{a}\cdot\vec{r})\vec{a} = \vec{0}$. 3

8. Evaluate $\int_C \vec{F}\cdot\overrightarrow{dr}$ where $\vec{F} = 3x^2y\,\hat{i} + (2y+1)\,\hat{j}$ and C is the curve joining (0,0) and (1,1) along $y=x^2$.

9. Verify Green's theorem for $\vec{F} = x^2\,\hat{i} + xy\,\hat{j}$, where R is the region bounded by $x^2 + \frac{y^2}{4} = 1$.

10. Use divergence theorem to evaluate $\iint_S \left(x^3\,dy\,dz + x^2y\,dz\,dx + x^2z\,dx\,dy\right)$, where S is the closed surface consisting of the cylinder $x^2+y^2=4 \ (0\le z\le 3)$ and the circular disc z = 0 and $z=3 \ \left(x^2+y^2\le 4\right)$. 5

Group-B

Answer <u>any four</u> questions from <u>question nos. 11 to 16</u> : [4×5]

11. Solve: $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+6y=0$.

12. Solve: $\left(D^2-1\right)y=e^{2x}$, where $D\equiv\dfrac{d}{dx}$.

13. Show that the differential equation $x^2\dfrac{d^2y}{dx^2}+6x\dfrac{dy}{dx}+8y=0$ may be reduced to a linear differential equation by the substitution $x=e^t$.

14. Consider the partial differential equation , $2u_{xx}-u_{xy}+3u_{yy}+u_x+2=0$.

i) Is it linear ? Justify .

ii) Is it parabolic ? Justify . (3+2)

15. The temperature T(x,t) in a stationary medium , $n\ge 0$ is governed by the heat conduction equation $\dfrac{\partial T}{\partial t}=\dfrac{\partial^2 T}{\partial x^2}$. Making the change of the variable $(x,t)\to(u,t)$, where $u=\dfrac{x}{2\sqrt{t}}$. Show that $4t\dfrac{\partial T}{\partial t}=\dfrac{\partial^2 T}{\partial u^2}+2u\dfrac{\partial T}{\partial u}$.

16. Consider the Laplace equation $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$, $y>0,-\infty<x<\infty$ $\dfrac{\partial u}{\partial y}(x,0)-u(x,0)=f(x)$, where $f(x)\in C_0^\infty\left(\mathbb{R}^1\right)$. Find a bounded solution u(x,y) and show that $u(x,y)\to 0$ where $|x|+y\to\infty$.

Group-C

Answer <u>any five</u> questions from <u>question nos. 17 to 24</u> : [5×5]

17. Define absolute and relative errors in Numerical Analysis. An approximate value of π is given by 3.1428571 and its true value is 3.1415926 . Find the absolute and relative errors . [2+3]

18. Using the properties of the shift operator E and backward difference operator ∇, prove that $\nabla\equiv 1-E^{-1}$ and $E\equiv e^{hD}$, where h>0 and $Dy(x)=\dfrac{d}{dx}y(x)$. [3+2]

19. Find the root of the polynomial $f(x)=x^3+x-1$, correct upto 5 decimal places , taking x=0.8 as a initial guess .

20. Find the root of $f(x) = x^3 - x - 1$ inside (1,2) , correct upto 3 decimal places using Bisection method.

21. Find Lagrange's interpolation polynomial of degree 2 by using the following table of values . Hence determine the value of y at x=2.7

x	2.0	2.5	3.0
f(*x*)	0.69315	0.91629	1.09861

[3+2]

22. Using the following table of values of x and y, calculate the values of y at x=0.40, correct to 4 decimal places.

x	0.10	0.15	0.20	0.25	0.30
y	0.1003	0.1511	0.2027	0.2553	0.3093

23. Using trapezoidal rule and the following table of values, calculate the values of $\int_0^1 y\,dx$, correct to 3 decimal places.

x	0	0.5	1.0
y	0	1.0	0.0

24. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$, using Simpson $\frac{1}{3}$ rule, correct upto 4 decimal places and taking 6 regular partition .

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